Delayed Feedback Control of Chaos in an Electronic Double-Scroll Oscillator *

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Z. Naturforsch. **49a,** 843–846 (1994); received May 30, 1994

We present experimental results on stabilizing unstable periodic orbits of an autonomous chaos oscillator based on a simple electronic circuit. Control is achieved by applying the difference between the actual and a delayed output signal of the oscillator. The quality of chaos control can be measured via the strength of perturbation. The dependence on the delay time shows a characteristic resonance-type behavior.

Since the pioneer work done by Hübler and coworkers [1], controlling chaos has become a fascinating topic in the field of nonlinear dynamics. In particular, the method introduced by Ott, Grebogi, and Yorke (OGY) [2] develops to an important tool to overcome undesired chaotic behavior in various fields of applications. The key observation is that the typical chaotic attractor embeds an infinite number of unstable periodic orbits of distinct periods T_i. They can be stabilized by an only small, carefully chosen, feedback perturbation applied to some system parameter available for external adjustment. The OGY approach is as follows: At first, one determines some of the unstable low-periodic orbits of the strange attractor by finding the fixed points in the Poincare map of the system. Then the linear transformations close to these points and their dependence on the parameter variation have to be reconstructed from experiment. Finally, a small time-dependent perturbation is applied to the system, in order to stabilize these already existing periodic orbits.

The OGY method is a very general one. It does not require any a priori analytical knowledge of the system dynamics and has been successfully applied to various physical experiments including a magnetic ribbon [3], a spin-wave system [4], a chemical system [5], an electric diode [6], laser systems [7], and cardiac systems [8].

* Paper presented at the 4th Annual Meeting of ENGADYN, Grenoble, October 11–14, 1993.

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Any experimental application of the OGY method requires permanent computer analysis of the state of the system. The changes of the parameter, however, are discrete in time, since the method deals with the Poincare map of the system. This makes the method touchy to noise. Small noise gives rise to occasional bursts of the system directing to the region far from the controlled periodic orbit. These bursts are more frequent for large noise [2]. Various modifications of the OGY method considered in [5–12] are also discrete in time. The details of these modifications can be found in a review paper [13].

Recently, one of us (K. P.) has suggested an alternative approach based on a time-continuous control [14–16]. The idea is to construct a time-continuous perturbation which does not change the desired periodic orbit, but only changes the corresponding Lyapunov exponents in such a way that the orbit becomes stable. Different types of perturbation satisfying this requirement can be considered. One approach is to use the conventional feedback loop [17] where the reference signal corresponds to the desired unstable periodic orbit [14, 15]. It can be reconstructed from the chaotic output signal by a standard method of delay coordinates. The same feedback loop can be used to stabilize unstable aperiodic orbits [16]. In this case, the reference signal simply represents the prerecorded history of the system. As a result, the current state of the system can be synchronized with its prerecorded history using only a small perturbation. An experimental application of this approach to an electronic double-scroll oscillator has been presented in [18]. Another approach to stabilize unstable periodic

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orbits is to use the delayed output signal of the system as a reference signal [14, 15]. The latter method deserves strong interest from an experimental point of view. It does not require any computer analysis of the state of the system and can be simply implemented in various experiments by a purely analog technique. First experimental applications to high-frequency nonautonomous chaos oscillators have been considered in [19] and [20].

In the present paper, we deal with the experimental realization of the delayed feedback control method to a low-frequency autonomous chaos oscillator. Although the low-frequency domain is not very handy for disposing the required delay line, it is well adapted to the observation and analysis of the experimental results. In particular, we have determined the dependence of the perturbation strength on the delay time which follows a characteristic resonance-type behavior.

The chaotic system to be controlled by the delayed feedback method is modeled by a set of ordinary differential equations [14]:

$$\dot{y} = P(y, \boldsymbol{x}) + F(t),$$
$$\dot{\boldsymbol{x}} = \boldsymbol{Q}(y, \boldsymbol{x}). \tag{1}$$

Imagine that the Eqs. (1) are unknown, but some scalar variable $y(t)$ can be measured as system output. Vector $\boldsymbol{x}$ describes the remaining variables of the system that are not available or not of interest for observation. $F(t)$ denotes an external perturbation which is chosen in the form of the difference between the actual and delayed output signal:

$$F(t) \equiv F(y(t-\tau), y(t)) = K\{y(t-\tau) - y(t)\}. \tag{2}$$

Here τ denotes the delay time and K the weight of the perturbation. The main feature of this perturbation is that it vanishes if the delay time τ coincides with the period T_i of some unstable periodic orbit of the unperturbed system: $F(t) \equiv 0$ at $\tau = T_i$. It means that, at this value of τ, the perturbation does not change the solution of (1) corresponding to the unstable orbit with the period T_i. Upon choosing an appropriate weight K of the feedback, one can achieve stabilization. Thus, two parameters τ and K have to be adjusted in experiment for stabilizing the desired periodic orbit. The amplitude of the feedback signal $F(t)$ can be considered as a criterion of the stabilization. When the system moves along its periodic orbit, this amplitude is extremely small. Its dependence on the delay time τ has to be of resonance type, with the minima at τ coinciding with the periods T_i of the distinct periodic orbits.

We have used an electronic autonomous chaos oscillator suggested by Shinriki et al. [21], in order to test the method at a real experimental situation. The circuit is shown in Fig. 1, while the equations of state are the following:

$$C_1 \dot{V}_1 = V_1\left(\frac{1}{R} - \frac{1}{R_1}\right) - f(V_1 - V_2) + I_c,$$
$$C_2 \dot{V}_2 = f(V_1 - V_2) - I_3, \tag{3}$$
$$L\dot{I}_3 = -I_3 R_3 + V_2,$$

where V_1, V_2, and I_3 are the voltage across the capacitor C_1, the voltage across the capacitor C_2, and the current through the inductor L, respectively. The negative impedance converter (NIC) consists of a standard operational amplifier (type TL071) biased symmetrically by ± 15 V, two feedback resistors, and a further resistor connected to the ground that gives the slope of the negative resistance. In the operation range of the voltage ΔV, it can be approximated by a linear negative resistance $-R$. The parallel resistor R_1 compensates the negative slope of the NIC and serves as a basic control parameter of the oscillator. The only existing nonlinearity in the system has its origin in two Zener diodes (3.3 V Z-diodes BZX 55C 3V3). Their current versus voltage characteristic can be approximated as follows:

$$I_d \equiv f_d(V) = \begin{cases} 0, & |V| < V_d; \\ \operatorname{sign}(V)\{A\Delta V^3 + B\Delta V^4 + C\Delta V^5\}, & |V| \geqq V_d, \end{cases} \tag{4}$$

where $V_d = 2.5$ V, $A = 2.2500$, $B = -1.9460$, $C = 0.8188$, $\Delta V = |V| - V_d$, and $\operatorname{sign}(V) = \pm 1$ for $V > 0$ and $V < 0$, respectively. The voltage V is given in volts, the current I_d in milliamps. The parallel resistor R_2 serves to vary the shunt of the capacitor C_1 and the resonant circuit LC_2:

$$I_N \equiv f(V) = f_d(V) + V/R_2. \tag{5}$$

In our experiment, we have fixed the parameters as follows: $C_1 = 10$ nF, $C_2 = 100$ nF, $L = 270$ mH, $R = 6.91$ kΩ, $R_2 = 15$ kΩ, $R_3 = 100$ Ω. R_1 was used as control parameter. Figure 2 shows the bifurcation diagram of the unperturbed ($I_c = 0$) system. With increasing R_1, the system behavior develops via the following scenario: Stable fixed point (I), period-doubling scenario (II), mono-scroll (Rössler type) chaos (III), dou-

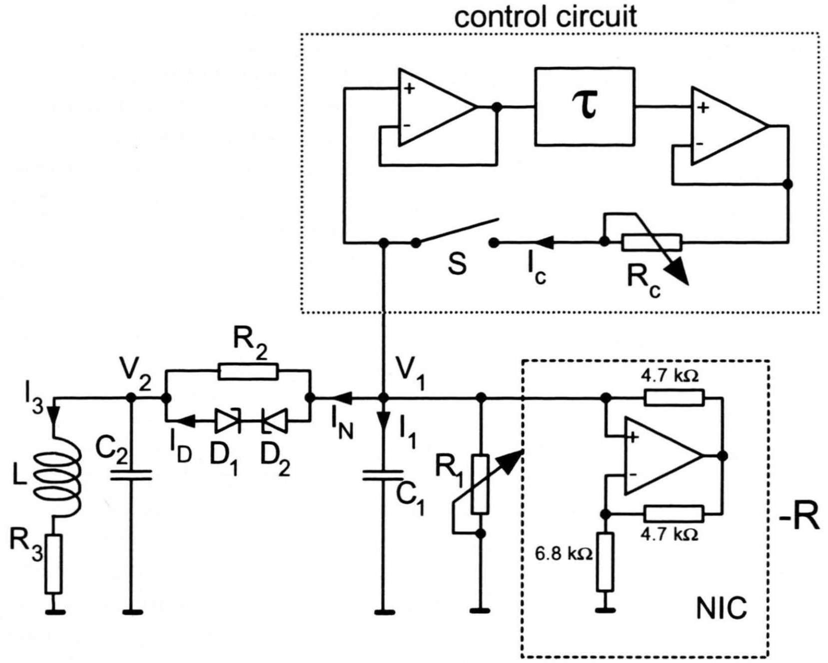

Fig. 1. Scheme of the nonlinear oscillator and the control circuit.

ble-scroll chaos (IV). The numerical bifurcation diagram obtained from the model equations (3) at the above values of the parameters is found to be in good qualitative agreement with the experimental one [18].

Control of the oscillator has been achieved by a simple control circuit shown in the upper right hand side of Figure 1. It first consists of an impedance converter for decoupling the delay line from the oscillator. There is a second impedance converter to feed the delayed signal back to the oscillator via the control resistor R_c. We use a commercially available, integrated "bucket brigade" delay line (National Panasonic, type MN3011) with a length of 396 stages. It allows to realize a variable delay time in the range from 0.8 to 20 ms. The delay time is adjusted by an external clock frequency generator. The control circuit branches off the control current in the form required by the method:

$$I_c = \frac{V_1(t-\tau) - V_1(t)}{R_c} \equiv K\,\{V_1(t-\tau) - V_1(t)\}. \qquad (6)$$

The weight of the perturbation K derives from the control resistor R_c ($K = 1/R_c$). The dependence of the control current on the delay time at a fixed value of R_c is shown in Figure 3. A deep minimum of the control current at a characteristic value of τ, coinciding with the period of the first unstable periodic orbit of the oscillator ($\tau = T_1 \approx 1.16$ ms), is observed. In this regime, the system produces strongly periodic oscillations corresponding to the first stabilized orbit. The control current is extremely small, $I_c \approx 2\,\mu$A (root mean

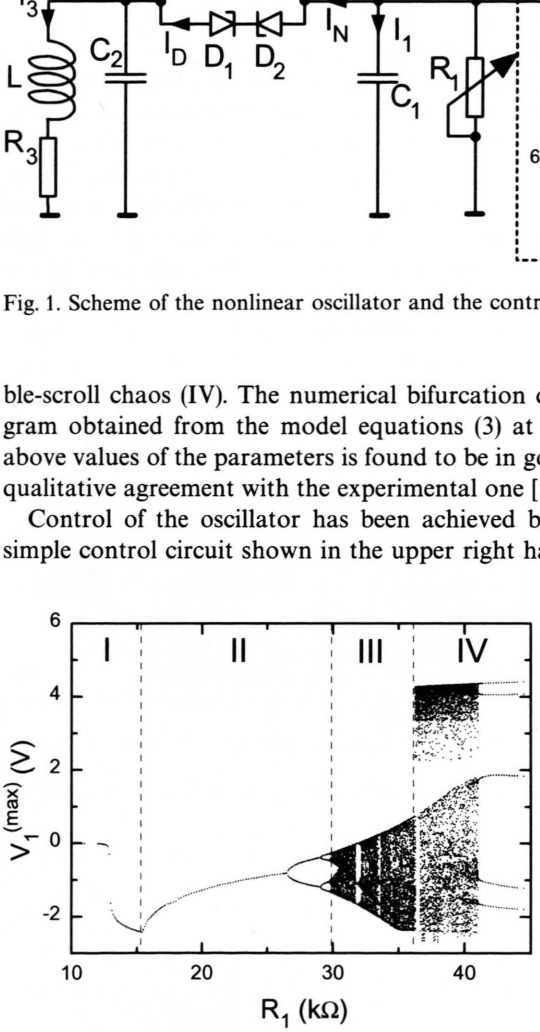

Fig. 2. Bifurcation diagram of the nonlinear oscillator. The distribution of the maxima of the voltage V_1 is plotted as a function of the control parameter R_1. Roman numerals mark distinct regimes: (I) stable fixed point, (II) period-doubling scenario, (III) mono-scroll chaos, (IV) double-scroll chaos.

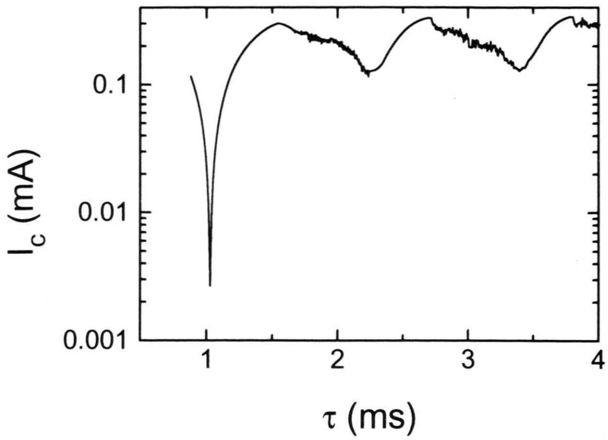

Fig. 3. Control current versus delay time of the controlled oscillator of Fig. 1 at $R_c = 10\ \text{k}\Omega$.

square value), compared with the total current I_1 through the capacitor C_1, $I_1 \approx 3$ mA. Therefore, stabilization of the first orbit is provided by a very small control signal with an amplitude of a relative value less than 0.1%.

With increasing τ, the control current again demonstrates resonance-type behavior at τ close to the period of the second ($\tau = T_2 \approx 2.25$ ms) and the third ($\tau = T_3 \approx 3.4$ ms) unstable periodic orbit of the system. However, these resonance minima are not pronounced. Strong stabilization of these orbits can not be achieved. The system behaves in an intermittent state. There are long time intervals where the control works and the system is locked into a periodic state. Then the system loses control and goes away from the periodic orbit. We observe alternating switching between these two states with the long periodic phase and occasional bursts into the region far from the controlled orbit.

In conclusion, we have implemented delayed feedback control to a low-frequency autonomous chaos oscillator. One advantage of the method is its simple experimental realization. It permits the stabilization of an unstable periodic orbit by an extremely small perturbation. The method does not require any computer analysis and any external reference signal. Control is achieved by autosynchronization of the actual state of the system with its time-delay state. The amplitude of the perturbation exhibits the resonance-type behavior when varying the delay time. The minima are observed at values of the delay time close to the periods of unstable periodic orbits. Such dependence can be considered as a powerful experimental tool of nonlinear spectroscopy, in order to analyze the unstable periodic states of chaotic systems.

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